Outline

- Mathematical model
- Image formation
  - time domain viewpoint
  - frequency domain viewpoint (for small scenes)
- Approximating targets by point clouds
- SAR interferometry
Mathematical Model

Maxwell’s equations $\rightarrow$ scalar wave equation

Green’s function + Born approximation

\[ p(t, x_r; x_s) \propto \int \rho(y) \frac{f''[t - \tau(y, x_s) - \tau(y, x_r)]}{|y - x_s||y - x_r|} \, dy \]

standard (monostatic) SAR:

\[ \tau(y, x) = \frac{|y - x|}{c_0} = \frac{|\gamma(s) - y|}{c_0} \]

\[ p(t, s) \propto \int \rho(y) \frac{f''(t - 2R_{s,y}/c_0)}{R_{s,y}^2} \, dy \]
figures from Brett Borden, Naval Postgraduate School
Image formation

\[ I(y) = \sum_{x_r, x_s} \text{data}(t = \tau(y, x_s) + \tau(y, x_r), x_r; x_s) \]

array imaging

\[ = \sum_s \text{data}(t = 2R_{s,y}/c_0, \gamma(s)) \]

standard (monostatic) SAR

why does this work?
Imaging from a single viewing position

cross range resolution $\Delta \theta$

range resolution $\Delta d$

arc with weak scattering

arc with strong scattering
Example with 3 scatterers

Imaging from a single view
Imaging from two views
Imaging from three views

Sensor Flight Path

synthetic aperture
Frequency domain viewpoint

time-domain model

\[ p(t, s) \propto \int \rho(y) \frac{f''(t - 2R_{s,y}/c_0)}{R^2_{s,y}} dy \]

Fourier transform in \( t \)

\[ \int \ldots e^{-i\omega t} dt \]

\[ P(\omega, s) \propto \int \rho(y) \frac{\omega^2 F(\omega)e^{-2i\omega R_{s,y}/c_0}}{R^2_{s,y}} dy \]

far-field approximation

\[ R_{s,y} = |\gamma(s) - y| \approx |\gamma| - \hat{\gamma} \cdot y + \ldots \quad |\gamma| \gg |y| \]

\[ \hat{\gamma} = \frac{\gamma}{|\gamma|} \]

\[ P(\omega, s) \propto \int \rho(y)e^{-2ik\hat{\gamma}(s) \cdot y} dy \]

to form image, invert Fourier transform!

\[ k = \frac{\omega}{c_0} \]
Approximating targets by point clouds

\[ P(\omega, s) \propto \int \rho(y) e^{-2ik\cdot y} dy \]

main contributions are from corners, edges, and specular points

k large -> use geometrical optics

\[ E_{\text{scatt}}(r) = \sum_{n=1}^{N} E_{\text{scatt}}^{(n)}(r) \]
Interferometry

\[ p(t, s) \propto \int \rho(y) \frac{f''(t - 2R_s, y/c_0)}{R^2_{s,y}} dy \]

narrowband

\[ f(t) = a(t)e^{i\omega_0 t} \quad a = \text{slowly varying (complex) amplitude} \]

\[ p(t, s) \propto \int \rho(y)e^{i\omega_0(t-2R_s,y/c_0)} \frac{a(t - 2R_s,y/c_0)}{R^2_{s,y}} dy \]

scattering takes place on surface

\[ \rho(y) = \tilde{\rho}(y_1, y_2)\delta(y_3 - h(y_1, y_2)) \]

\[ y = y_T + h(y_T)\hat{e}_3 \quad y_T = (y_1, y_2, 0) \]

\[ R_{s,y} = |y_T + h\hat{e}_3 - \gamma| = |y_T - \gamma| + h(y_T)\hat{e}_3 \cdot \overrightarrow{y_T - \gamma} \]

\[ p(t, s) \approx \int \left[ \tilde{\rho}(y_T)e^{2ik_0d(y_T)} \right] e^{i\omega_0(t-R_{s,y_T}/c_0)} \frac{a(t - R_{s,y_T}/c_0)}{R^2_{s,y_T}} dy_T \]

target phase encodes height information!